

The 1st quiz in Pattern Recognition and Machine Learning

Due 19 June 2023 (in the beginning of the lecture)

Exercises

Exercise 2.1 Probabilities are sensitive to the form of the question that was used to generate the answer

(Source: Minka.) My neighbor has two children. Assuming that the gender of a child is like a coin flip, it is most likely, a priori, that my neighbor has one boy and one girl, with probability $1/2$. The other possibilities—two boys or two girls—have probabilities $1/4$ and $1/4$.

- Suppose I ask him whether he has any boys, and he says yes. What is the probability that one child is a girl?
- Suppose instead that I happen to see one of his children run by, and it is a boy. What is the probability that the other child is a girl?

Exercise 2.2 Legal reasoning

(Source: Peter Lee.) Suppose a crime has been committed. Blood is found at the scene for which there is no innocent explanation. It is of a type which is present in 1% of the population.

- The prosecutor claims: “There is a 1% chance that the defendant would have the crime blood type if he were innocent. Thus there is a 99% chance that he guilty”. This is known as the **prosecutor’s fallacy**. What is wrong with this argument?
- The defender claims: “The crime occurred in a city of 800,000 people. The blood type would be found in approximately 8000 people. The evidence has provided a probability of just 1 in 8000 that the defendant is guilty, and thus has no relevance.” This is known as the **defender’s fallacy**. What is wrong with this argument?

Exercise 2.3 Variance of a sum

Show that the variance of a sum is $\text{var}[X + Y] = \text{var}[X] + \text{var}[Y] + 2\text{cov}[X, Y]$, where $\text{cov}[X, Y]$ is the covariance between X and Y

Exercise 2.5 The Monty Hall problem

(Source: Mackay.) On a game show, a contestant is told the rules as follows:

There are three doors, labelled 1, 2, 3. A single prize has been hidden behind one of them. You get to select one door. Initially your chosen door will *not* be opened. Instead, the gameshow host will open one of the other two doors, and *he will do so in such a way as not to reveal the prize*. For example, if you first choose door 1, he will then open one of doors 2 and 3, and it is guaranteed that he will choose which one to open so that the prize will not be revealed.

At this point, you will be given a fresh choice of door: you can either stick with your first choice, or you can switch to the other closed door. All the doors will then be opened and you will receive whatever is behind your final choice of door.

Imagine that the contestant chooses door 1 first; then the gameshow host opens door 3, revealing nothing behind the door, as promised. Should the contestant (a) stick with door 1, or (b) switch to door 2, or (c) does it make no difference? You may assume that initially, the prize is equally likely to be behind any of the 3 doors. Hint: use Bayes rule.

Exercise 2.6 Conditional independence

(Source: Koller.)

- a. Let $H \in \{1, \dots, K\}$ be a discrete random variable, and let e_1 and e_2 be the observed values of two other random variables E_1 and E_2 . Suppose we wish to calculate the vector

$$\vec{P}(H|e_1, e_2) = (P(H = 1|e_1, e_2), \dots, P(H = K|e_1, e_2))$$

Which of the following sets of numbers are sufficient for the calculation?

- i. $P(e_1, e_2), P(H), P(e_1|H), P(e_2|H)$
 - ii. $P(e_1, e_2), P(H), P(e_1, e_2|H)$
 - iii. $P(e_1|H), P(e_2|H), P(H)$
- b. Now suppose we now assume $E_1 \perp E_2|H$ (i.e., E_1 and E_2 are conditionally independent given H). Which of the above 3 sets are sufficient now?

Show your calculations as well as giving the final result. Hint: use Bayes rule.

Exercise M1 (Mine's problem)

Write a program using your favorite language to calculate the volume of a D-dimensional unit sphere by random sampling of N points, that is, by Monte Carlo simulation. Draw graphs to illustrate the results, taking the value of D in the x-axis, for D=1-20 and N=100,1000,10000. Submit only the graph, no programing source file.